

Advanced Algebra Study Guide

An Engineering Approach to Learning, Solving, Understanding and
Teaching Algebra using Formulas Graphs and Diagrams.

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0.1 Appreciation

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Prefix

0.2 Introduction

After a career in Electrical and Computer Test engineering, then teaching math in college I compiled much of my notes used in the classroom to help future students gain math instinctive problem-solving skills, using the tools presented in class.

0.3 Scope of this Book

The Scope of this book is to provide a study guide for many of the principles covered in my College Algebra Courses.

0.4 Intention for Results of the study guide

The result of this study is intended to give the student a better understanding of a solid approach to solving math Problems. In subsequent tests and in the Engineering field the student may need to re-derive the principle or equation to solve the problem presented by the required task. The student learns to use underlying principles to create mathematical flexibility to solve problems in the class room or engineering tasks in the workplace.

0.5 My Math website: Mathstudyguide.org

Please feel free to check out my website: mathstudyguide.org

Chapter 1

Algebra Overview

The basic mode of algebra is to develop rules for handling unknowns, coefficients and numbers and exponents.

1.1 Letter Variables

The basic letter variables of Algebra are usually (x , y , and z) and possibly (a , b , and c)

Any letter can be used depending on the application required.

1.2 State of variables during an Algebra Problem

The Algebraic variable is constant for the length of the solution.

Note: In calculus the variables are (v , t , d , and a) constantly changing during the problem.

For instance, a drag race car if accelerating at a constant rate the velocity is continually changing. The time and distance are changing as well.

1.3 General Characteristics of Algebra

Algebra is a Math that can transform a formula from a word problem which is composed of add terms and subtracted terms into factors of multiplies. When in the multiplied factors format, we can solve the quadratic equation

by setting the equation to either y (for the graph) or zero (for the x intercepts or roots).

$x^2 - 4x - 96 = 0$	Solve for x -intersection (solution)
$x^2 - 4x - 96 = y$	Graph on a graphing calculator

$96 = x(x - 4)$	Original Equation
$96 = x^2 - 4x$	Original Equation adds and subtracts
$(x - 12)(x + 8) = 0$	Factored form (factors); can be solved

Refer to page 104 for the solution.

1.4 Cross reference to solving an equation by factoring

Another example of a quadratic problem solution can be found on page 115

Example: Calculate the time the ball takes to hit the ground when thrown upwards from a certain height.

1.5 Zero-Factor property, rule

The Zero-Factor Rule, Explanation is found on page 22

We can use the zero-factor rule.

When there two factors equal to zero then either factor or both are equal to zero.

Therefore, set each factor equal to zero and solve.

$(x + 1)(x - 7) = 0$ By the zero-factor property
set each factor equal to zero

$$\begin{aligned}(x + 1) &= 0; & x &= -1 \\(x - 7) &= 0; & x &= 7\end{aligned}$$

1.6 Variables in a problem Algebra Problem using the same formula

In a further example the constants representing the variable will change while the formula will be reused for different initial and final conditions. In other words, the formula can be re-used with different constants but using the same variables without changing the equation. In summary in algebra the attribute is that the variable stays the same.